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Backward waves and negative refractive indices in gyrotropic chiral media

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Abstract

The possibility of the backward waves and negative refractive indices of the gyrotropic chiral materials is studied, and the impedances of the eigenmodes are derived. Since the gyrotropic parameters in the permittivity and permeability tensors favour the realization of the negative refractive index in the gyrotropic chiral material, the negatively refracting medium can be achieved even far off the resonances of the permittivity and permeability. A potential effect of the field quantization in a compact subwavelength cavity resonator containing the gyrotropic chiral material is suggested.

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1. Introduction

Recently, a new type of artificial metamaterials (also called *left-handed media* or *negative refractive index media*), whose electric permittivity and magnetic permeability are simultaneously negative in a frequency band, have attracted considerable attention of many researchers in various fields (see e.g. [1–3]). These metamaterials exhibit a number of peculiar electromagnetic and optical properties, including the reversals of both Doppler shift and Cherenkov radiation [1], negative refraction [1], amplification of evanescent waves [3] (which leads to subwavelength focusing (see e.g. [3, 4])), negative Goos-Hänchen shift [5], reversed circular Bragg phenomenon [6] and localization of electromagnetic waves [7]. One of the potential applications of negative refractive index materials is the so-called superlens (perfect lens) because a slab of such a metamaterial may focus both the propagating and evanescent components of the field produced by a point source (object) [3]. At present, there are several approaches to the realization of negative index materials, including artificial metamaterials

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[8, 9]), photonic crystal structures [10–13], transmission line simulation [14] as well as photonic resonant materials [15, 16]. The first three methods are based on the classical electromagnetic theory, and the mechanism in the fourth scheme is the quantum coherence and quantum interference.

Left-handed media in the microwave region have been fabricated experimentally [2]. The impact would be much larger if we can realize negative refraction at optical frequencies (beating the diffraction limit can give revolutionary breakthroughs to, e.g., the optical storage industries). More recently, Tretyakov and Pendry *et al* suggested a new route to the negative refraction [17, 18] by using chiral materials, the permittivity of which can have resonances in the optical frequency regions. Some natural and optically chiral media can be considered as a homogeneous medium (sugar solution in water is probably the cheapest optical chiral medium). Theoretically, a chiral parameter can actually be larger than the square root of the product of permittivity and permeability, and then negative refraction (or backward wave) will occur at one of the eigen polarizations [19]. On the other hand, the chiral parameter is always small for all the existing chiral media (regardless of natural or artificial). Therefore, a good way to achieve negative refraction in a chiral medium is to choose a working frequency at which permittivity is very small. This can occur at or near the resonant frequency of the permittivity of a chiral medium (called chiral nihility). But the chiral nihility does not arise when the frequency is far off resonance. In the present paper, we study the possibility of backward wave propagation in a generalized material (gyrotropic chiral medium). We think the gyrotropic chiral medium may have some advantage over the chiral medium: specifically, negative refraction can occur in a gyrotropic chiral medium without requiring the permittivity to be very small at a working frequency (cf equation (21)). In other words, the negative refractive index and backward wave propagation can be achieved even far off the resonances of the permittivity and permeability because the gyrotropic parameters can dramatically reduce the refractive indices of the eigenmodes inside the gyrotropic chiral medium. This is one of the most remarkable features in the present scheme to realize negative refraction[3]. We also study the impedance of the polarized modes inside the gyrotropic chiral media, and consider the possibility of impedance match at the air–medium interfaces. In addition, we suggest that a field quantization effect may arise in the subwavelength cavity resonator containing the gyrotropic chiral medium.

2. Backward eigenmodes in a gyrotropic chiral medium

The constitutive relation of gyrotropic media involves the permittivity and permeability tensors $\hat{\epsilon}$, $\hat{\mu}$. The constitutive relation of a gyrotropic chiral medium can be written in the following general form:

$$\begin{cases} \mathbf{D} = \hat{\epsilon}\epsilon_0\mathbf{E} + (\chi + \mathrm{j}\alpha)\mathbf{H}, \\ \mathbf{B} = \hat{\mu}\mu_0\mathbf{H} + (\chi - \mathrm{j}\alpha)\mathbf{E}, \end{cases} \tag{1}$$

where α and χ denote the chirality and nonreciprocity parameters, respectively [20]. j is the imaginary unit, satisfying $\mathrm{j}^2 = -1$. The permittivity and permeability tensors are [1]

$$\hat{\epsilon} = \begin{pmatrix} \epsilon_1 & -\mathrm{j}\epsilon_2 & 0 \\ \mathrm{j}\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}, \qquad \hat{\mu} = \begin{pmatrix} \mu_1 & -\mathrm{j}\mu_2 & 0 \\ \mathrm{j}\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}, \tag{2}$$

[3] After this paper was accepted, we found that very recently, Mackay and Lakhtakia had considered a scheme of negative refraction similar to ours but with different emphases (quantum or classical aspects) on the electromagnetic and optical properties [28].

where ϵ_2 and μ_2 are the gyroelectric and gyromagnetic parameters, respectively. Such a gyrotropic chiral medium can be a chiroplasma material consisting of chiral objects embedded in a magnetically biased plasma, or a chiroferrite material made from chiral objects immersed in a magnetically biased ferrite [21].

In what follows, we analyse the wave propagation inside the gyrotropic chiral medium, and study the refractive indices for the eigenmodes inside the medium. Consider a time harmonic wave $\exp[\mathrm{j}(\omega t - kx_3)]$, which is propagating along the x_3-direction inside the gyrotropic chiral media. From equations (1) and (2), one can have

$$\mathbf{D} = \begin{pmatrix} \epsilon_0(\epsilon_1 E_1 - \mathrm{j}\epsilon_2 E_2) + (\chi + \mathrm{j}\alpha)H_1 \\ \epsilon_0(\mathrm{j}\epsilon_2 E_1 + \epsilon_1 E_2) + (\chi + \mathrm{j}\alpha)H_2 \\ 0 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} \mu_0(\mu_1 H_1 - \mathrm{j}\mu_2 H_2) + (\chi - \mathrm{j}\alpha)E_1 \\ \mu_0(\mathrm{j}\mu_2 H_1 + \mu_1 H_2) + (\chi - \mathrm{j}\alpha)E_2 \\ 0 \end{pmatrix}. \tag{3}$$

For a time harmonic electromagnetic wave, the Maxwellian equations $\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$, $\nabla \times \mathbf{H} = \frac{\partial}{\partial t}\mathbf{D}$ can be rewritten as $\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$, $\mathbf{k} \times \mathbf{H} = -\omega \mathbf{D}$. In the meantime, we can obtain the relations $\mathbf{k} \times \mathbf{E} = (-kE_2, kE_1, 0)$, $\mathbf{k} \times \mathbf{H} = (-kH_2, kH_1, 0)$ for the time harmonic wave $\exp[\mathrm{j}(\omega t - kx_3)]$ propagating along the x_3-direction.

In view of the above analysis, the following relations can be obtained:

$$\begin{cases} -kE_2 = \omega[\mu_0(\mu_1 H_1 - \mathrm{j}\mu_2 H_2) + (\chi - \mathrm{j}\alpha)E_1], \\ kE_1 = \omega[\mu_0(\mathrm{j}\mu_2 H_1 + \mu_1 H_2) + (\chi - \mathrm{j}\alpha)E_2], \\ -kH_2 = -\omega[\epsilon_0(\epsilon_1 E_1 - \mathrm{j}\epsilon_2 E_2) + (\chi + \mathrm{j}\alpha)H_1], \\ kH_1 = -\omega[\epsilon_0(\mathrm{j}\epsilon_2 E_1 + \epsilon_1 E_2) + (\chi + \mathrm{j}\alpha)H_2]. \end{cases} \tag{4}$$

From the first and second formulae in equation (4), we can obtain the expressions for E_2 and E_1 in terms of both H_2 and H_1:

$$E_2 = \frac{-1}{k^2 + (\chi - \mathrm{j}\alpha)^2\omega^2}\{[\omega k\mu_0\mu_1 + \mathrm{j}(\chi - \mathrm{j}\alpha)\omega^2\mu_0\mu_2]H_1 + [-\mathrm{j}\omega k\mu_0\mu_2 + (\chi - \mathrm{j}\alpha)\omega^2\mu_0\mu_1]H_2\}, \tag{5}$$

and

$$E_1 = \frac{1}{k^2 + (\chi - \mathrm{j}\alpha)^2\omega^2}\{[\mathrm{j}\omega k\mu_0\mu_2 - (\chi - \mathrm{j}\alpha)\omega^2\mu_0\mu_1]H_1 + [\omega k\mu_0\mu_1 + \mathrm{j}(\chi - \mathrm{j}\alpha)\omega^2\mu_0\mu_2]H_2\}. \tag{6}$$

For convenience, expressions (5) and (6) are rewritten as

$$E_1 = bH_1 - aH_2, \qquad E_2 = aH_1 + bH_2, \tag{7}$$

where

$$\begin{cases} a = -\dfrac{1}{k^2 + (\chi - \mathrm{j}\alpha)^2\omega^2}[\omega k\mu_0\mu_1 + \mathrm{j}(\chi - \mathrm{j}\alpha)\omega^2\mu_0\mu_2], \\ b = -\dfrac{1}{k^2 + (\chi - \mathrm{j}\alpha)^2\omega^2}[-\mathrm{j}\omega k\mu_0\mu_2 + (\chi - \mathrm{j}\alpha)\omega^2\mu_0\mu_1]. \end{cases} \tag{8}$$

We analyse the wave vectors of the time harmonic wave propagating inside the gyrotropic chiral medium. Substitution of expression (7) into the third and fourth formulae of equation (4) yields

$$\begin{cases} \left(\dfrac{k}{\omega} + \epsilon_0\epsilon_1 a + \mathrm{j}\epsilon_0\epsilon_2 b\right)H_2 = [\epsilon_0(\epsilon_1 b - \mathrm{j}\epsilon_2 a) + (\chi + \mathrm{j}\alpha)]H_1, \\ \left(-\dfrac{k}{\omega} - \mathrm{j}\epsilon_0\epsilon_2 b - \epsilon_0\epsilon_1 a\right)H_1 = [\epsilon_0(-\mathrm{j}\epsilon_2 a + \epsilon_1 b) + (\chi + \mathrm{j}\alpha)]H_2, \end{cases} \tag{9}$$

which leads to the following relation:

$$\left(\frac{k}{\omega}+\epsilon_0\epsilon_1 a+\mathrm{j}\epsilon_0\epsilon_2 b\right)^2+[\epsilon_0(\epsilon_1 b-\mathrm{j}\epsilon_2 a)+(\chi+\mathrm{j}\alpha)]^2=0. \tag{10}$$

Further calculation yields the two expressions (dispersion relations):

$$\frac{k}{\omega}+\epsilon_0\epsilon_1 a+\mathrm{j}\epsilon_0\epsilon_2 b=\mathrm{j}[\epsilon_0(\epsilon_1 b-\mathrm{j}\epsilon_2 a)+(\chi+\mathrm{j}\alpha)], \tag{11}$$

and

$$\frac{k}{\omega}+\epsilon_0\epsilon_1 a+\mathrm{j}\epsilon_0\epsilon_2 b=-\mathrm{j}[\epsilon_0(\epsilon_1 b-\mathrm{j}\epsilon_2 a)+(\chi+\mathrm{j}\alpha)]. \tag{12}$$

We first consider the dispersion relation (11), which can now be rewritten in the form

$$\frac{k}{\omega}+\epsilon_0(\epsilon_1-\epsilon_2)(a-\mathrm{j}b)=\mathrm{j}(\chi+\mathrm{j}\alpha), \tag{13}$$

where the explicit expression for the term $a-\mathrm{j}b$ can be derived from the definition (8), i.e.,

$$a-\mathrm{j}b=-\frac{\omega\mu_0(\mu_1-\mu_2)}{k+\mathrm{j}\omega(\chi-\mathrm{j}\alpha)}. \tag{14}$$

Inserting expression (8) for the parameters a, b into formulae (13) and (14), one can obtain

$$\left(\frac{k}{\omega}+\alpha\right)^2=\epsilon_0\mu_0(\epsilon_1-\epsilon_2)(\mu_1-\mu_2)-\chi^2, \tag{15}$$

and hence

$$\frac{k}{\omega}=\pm\sqrt{\epsilon_0\mu_0(\epsilon_1-\epsilon_2)(\mu_1-\mu_2)-\chi^2}-\alpha. \tag{16}$$

Secondly, we consider another dispersion relation (12), which can be rewritten as

$$\frac{k}{\omega}+\epsilon_0(\epsilon_1+\epsilon_2)(a+\mathrm{j}b)=-\mathrm{j}(\chi+\mathrm{j}\alpha), \tag{17}$$

where

$$a+\mathrm{j}b=-\frac{\omega\mu_0(\mu_1+\mu_2)}{k-\mathrm{j}\omega(\chi-\mathrm{j}\alpha)}. \tag{18}$$

Inserting expression (8) for the parameters a, b into formulae (17) and (18), one can obtain

$$\left(\frac{k}{\omega}-\alpha\right)^2=\epsilon_0\mu_0(\epsilon_1+\epsilon_2)(\mu_1+\mu_2)-\chi^2, \tag{19}$$

and hence

$$\frac{k}{\omega}=\pm\sqrt{\epsilon_0\mu_0(\epsilon_1+\epsilon_2)(\mu_1+\mu_2)-\chi^2}+\alpha. \tag{20}$$

The four roots in expressions (16) and (20) correspond to the four eigenmodes (i.e., two pairs of counter-propagating modes for two mutually perpendicular polarization vectors $\mathrm{p}_\mathrm{R}, \mathrm{p}_\mathrm{L}$) inside the gyrotropic chiral material. According to the two final results (16) and (20), the refractive indices for the time harmonic waves inside the gyrotropic chiral medium are

$$\begin{aligned} n_\mathrm{R}&=c[\sqrt{\epsilon_0\mu_0(\epsilon_1\pm\epsilon_2)(\mu_1\pm\mu_2)-\chi^2}+\alpha],\\ n_\mathrm{L}&=c[\sqrt{\epsilon_0\mu_0(\epsilon_1\pm\epsilon_2)(\mu_1\pm\mu_2)-\chi^2}-\alpha], \end{aligned} \tag{21}$$

for the polarization vectors $\mathrm{p}_\mathrm{R}, \mathrm{p}_\mathrm{L}$, respectively. The signs $\pm$ for each polarization correspond to the waves propagating along the $\pm x_3$-directions.

Here we discuss the possibility of backward eigenmodes in a gyrotropic chiral medium through analysing expression (21).

(i) The case of pure chirality (without any gyrotropy): $\mu_2 = \epsilon_2 = 0$, and $\chi = 0$. Expression (21) becomes

$$n_{\mathrm{R,L}} = c(\sqrt{\epsilon_0\mu_0\epsilon_1\mu_1} \pm \alpha). \tag{22}$$

This is the so-called chiral route to negative refractive index materials (the case considered by Tretyakov *et al* [19, 18]). Recent studies have demonstrated that the backward waves can propagate in chiral materials with positive parameters ϵ_1, μ_1 [19, 18]. Near the resonance of electric and magnetic susceptibilities $\epsilon_1\mu_1$ can become smaller. If $\epsilon_1\mu_1 = 0$ (nihility) or $\epsilon_0\mu_0\epsilon_1\mu_1 < \alpha^2$, one of the refractive indices $n_{\mathrm{R,L}}$ will possess a minus sign, which means that a backward wave can propagate in such chiral media. As stated by Tretyakov recently, the above phenomenon in chiral microwave composites can take place naturally, because the inclusions are usually small spirals having resonant electric and magnetic polarizabilities [17]. As the chiral media are readily available and the permittivity can have resonances in the optical range, the chiral nihility is a very exciting new opportunity to realize negative refraction and related effects in the optical region in effectively uniform media [17]. During the last two decades, chiral media and bi-isotropic materials have captured extensive attention of many authors. But no one has paid attention to the possible backward wave propagation in chiral media. Instead, they believed that both of the two eigenmodes should be forward, and formulated a corresponding restriction $\epsilon_0\mu_0\epsilon_1\mu_1 > \alpha^2$ for the material parameters [23]. In fact, this restriction condition may be neither necessary nor essential for the material parameters and can be ruled out without any fear [17]. This, therefore, allows the backward wave propagation in chiral media. The chiral parameter can actually be larger than the square root of the product of permittivity and permeability, and then negative refraction (or backward wave) will occur at one of the eigen polarizations [19].

In a word, chiral negative refraction is a new way to realize the backward wave propagation under the condition that the permittivity is very small at a working frequency. But we will show later that the negative refraction can also occur in a gyrotropic chiral medium without requiring the permittivity to be very small at a working frequency.

(ii) The case of general bi-isotropic media: $\mu_2 = \epsilon_2 = 0$, but $\chi \neq 0$. Expression (21) becomes

$$n_{\mathrm{R,L}} = c(\sqrt{\epsilon_0\mu_0\epsilon_1\mu_1 - \chi^2} \pm \alpha). \tag{23}$$

If the nonreciprocity parameter χ is large enough, the Tellegen medium may have an opportunity to realize a negative index medium that does not require that the permittivity is very small at the working frequency. However, the nonreciprocity parameters χ for nearly all the natural and artificial materials are very small (or negligibly small); it is thus not practical to realize the negative refractive index by fabricating a Tellegen medium. We will not consider this case further in this paper.

(iii) The case of gyrotropic chiral media: $\chi = 0$. In this case expression (21) becomes

$$\begin{aligned} n_{\mathrm{R}} &= c[\sqrt{\epsilon_0\mu_0(\epsilon_1 \pm \epsilon_2)(\mu_1 \pm \mu_2)} + \alpha], \\ n_{\mathrm{L}} &= c[\sqrt{\epsilon_0\mu_0(\epsilon_1 \pm \epsilon_2)(\mu_1 \pm \mu_2)} - \alpha]. \end{aligned} \tag{24}$$

It follows that the negative index of refraction may be easily achieved even for a small chiral parameter α since we can let $\sqrt{\epsilon_0\mu_0(\epsilon_1 \pm \epsilon_2)(\mu_1 \pm \mu_2)} \simeq 0$ if one of the relations $\epsilon_1 \simeq \pm\epsilon_2$, $\mu_1 \simeq \pm\mu_2$ is satisfied. Recently, Jonsson *et al* proposed a theory of parametric generation and amplification for the off-diagonal gyrotropic-matrix elements in an artificially gyrotropic medium [22]. We believed that the relation $\epsilon_1 \simeq \pm\epsilon_2$ or $\mu_1 \simeq \pm\mu_2$ can be realized in the near future with some modern technology, and that the negative refraction in a gyrotropic chiral medium can be obtained at a far off resonant frequency.

3. Impedances of the backward eigenmode

As shown in [23], the chiral material can be described as the equivalent isotropic media with respective $\epsilon_\pm$ and $\mu_\pm$. In this formulation, the electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ in chiral media can be rewritten as $\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_-$ and $\mathbf{H} = \mathbf{H}_+ + \mathbf{H}_-$, respectively. Here $\mathbf{E}_\pm$ and $\mathbf{H}_\pm$ are called wavefields [23]. We use the concept of *wavefields* to study the impedances of the eigenmodes inside the gyrotropic chiral materials. This requires that the field quantities $\mathbf{D}_\pm, \mathbf{B}_\pm$ of the eigenmodes can be written in the form

$$\begin{cases} \mathbf{D}_+ = \hat{\epsilon}\epsilon_0\mathbf{E}_+ + \mathrm{j}\alpha\mathbf{H}_+ = \epsilon_+\epsilon_0\mathbf{E}_+, \\ \mathbf{B}_+ = \hat{\mu}\mu_0\mathbf{H}_+ - \mathrm{j}\alpha\mathbf{E}_+ = \mu_+\mu_0\mathbf{H}_+, \\ \mathbf{D}_- = \hat{\epsilon}\epsilon_0\mathbf{E}_- + \mathrm{j}\alpha\mathbf{H}_- = \epsilon_-\epsilon_0\mathbf{E}_-, \\ \mathbf{B}_- = \hat{\mu}\mu_0\mathbf{H}_- - \mathrm{j}\alpha\mathbf{E}_- = \mu_-\mu_0\mathbf{H}_-. \end{cases} \tag{25}$$

It can be verified that the respective permittivities $\epsilon_\pm$ and permeabilities $\mu_\pm$ of the eigenmodes should agree with the following relation:

$$\mu_0\epsilon_0(\hat{\mu} - \mu_\pm)(\hat{\epsilon} - \epsilon_\pm) = \alpha^2. \tag{26}$$

Assuming that the wavefields $\mathbf{E}_\pm$ and $\mathbf{H}_\pm$ satisfy the Maxwellian equations $\nabla \times \mathbf{E}_\pm + \mathrm{j}\omega\mu_\pm\mu_0\mathbf{H}_\pm = 0, \nabla \times \mathbf{H}_\pm - \mathrm{j}\omega\epsilon_\pm\epsilon_0\mathbf{E}_\pm = 0$, we can then obtain the permittivity $\epsilon_\pm$ and permeability $\mu_\pm$ of the eigenmodes as follows:

$$\begin{cases} \epsilon_\pm = \sqrt{\dfrac{\epsilon_1 \pm \epsilon_2}{\mu_1 \pm \mu_2}}\left[\sqrt{(\epsilon_1 \pm \epsilon_2)(\mu_1 \pm \mu_2)} \pm \dfrac{\alpha}{\sqrt{\epsilon_0\mu_0}}\right], \\ \mu_\pm = \sqrt{\dfrac{\mu_1 \pm \mu_2}{\epsilon_1 \pm \epsilon_2}}\left[\sqrt{(\epsilon_1 \pm \epsilon_2)(\mu_1 \pm \mu_2)} \pm \dfrac{\alpha}{\sqrt{\epsilon_0\mu_0}}\right], \end{cases} \tag{27}$$

and hence the impedances of the eigenmodes are (in the unit of vacuum impedance η_0)

$$\eta_\pm \equiv \sqrt{\frac{\mu_\pm}{\epsilon_\pm}} = \sqrt{\frac{\mu_1 \pm \mu_2}{\epsilon_1 \pm \epsilon_2}}. \tag{28}$$

The negatively refracting material can be utilized to fabricate a perfect lens [3]. In order to design perfect lenses, a good impedance match at the vacuum–medium interfaces is required. From the above expression one sees that it is possible to match the impedance $\eta_- = \sqrt{\frac{\mu_1-\mu_2}{\epsilon_1-\epsilon_2}}$ for the backward eigenmode to that of air at a certain frequency.

4. Subwavelength cavity resonator containing gyrotropic chiral materials

Apart from the superlenses, one of the most exciting applications of the negative refractive index materials is the so-called compact thin subwavelength cavity resonators (figure 1) [19, 24, 25]. We study the possibility of the field quantization effect in the compact cavity resonator that is formed by a layer of gyrotropic chiral material and a layer of a conventional dielectric medium (see figure 1). It was shown that a pair of planar waves travelling inside the dielectric system of two planar slabs positioned between two metal planes (or perfectly conducting reflectors) can satisfy the boundary conditions on the walls (reflectors) and on the interface between the two slabs even with very thin layers, provided that one of the slabs has a negative refractive index [19]. In [19, 24, 25], the authors showed that the total thickness

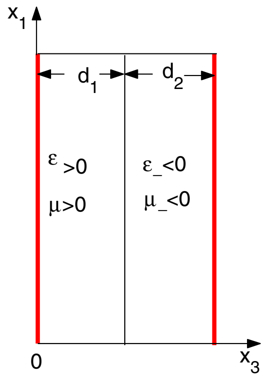

Figure 1. Configuration of a 1D subwavelength cavity resonator formed by a conventional medium layer on the left-hand side and a gyrotropic chiral medium layer on the right-hand side between two electrically perfectly conducting reflectors at $z = 0$ and $z = d_1 + d_2$.

can be far less than the resonant wavelength in the cavity resonator if the thickness ratio of the two layers is chosen appropriately. The cavity resonator containing a layer of gyroelectric chiral material with a negative refractive index may have two features: (i) compactness, (ii) nearly zero value for $(\epsilon_1 - \epsilon_2)$. These two features may lead to a quantum field effect inside the subwavelength resonator. For simplicity, in the following analysis we consider only the eigenmode with the equivalent permittivity ϵ_- and permeability μ_- for the gyrotropic chiral medium [23].

According to the second quantization formulation, the field operator of the planar wave electric field is given by

$$\hat{\mathbf{E}} = \mathrm{i}\sqrt{\frac{\hbar\omega\mu}{4\epsilon_0 n\gamma V}}\left[\hat{a}_{\mathbf{k}}\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r}) - \hat{a}_{\mathbf{k}}^{\dagger}\exp(-\mathrm{i}\mathbf{k}\cdot\mathbf{r})\right]\mathbf{e}_{\mathbf{k}}, \tag{29}$$

where n and μ denote the relative refractive index and the relative permeability, respectively, and $\gamma = \mathrm{d}(n\omega)/\mathrm{d}\omega$, V is the volume of the medium, and $\mathbf{e}_{\mathbf{k}}$ stands for the real unit polarization vector orthogonal to the wave vector $\mathbf{k}$ (we assume that the wave vector $\mathbf{k}$ is parallel to the x_3-direction in figure 1). Here we use a coherent state to compare a classical electromagnetic field with a quantized photon field. The coherent state is the most important field state because it mimics a classical field. The coherent state of an electromagnetic field is defined by

$$|\alpha_{\mathbf{k}}\rangle \equiv \mathcal{D}(\alpha_{\mathbf{k}})|0\rangle = \exp\left(\alpha_{\mathbf{k}}\hat{a}_{\mathbf{k}}^{\dagger} - \alpha_{\mathbf{k}}^{*}\hat{a}_{\mathbf{k}}\right)|0\rangle, \tag{30}$$

where $\hat{a}_{\mathbf{k}}^{\dagger}$, $\hat{a}_{\mathbf{k}}$ denote the photon creation and annihilation operators, respectively. $|0\rangle$ is the vacuum state. The coherent state has the following properties: $\hat{a}_{\mathbf{k}}|\alpha_{\mathbf{k}}\rangle = \alpha_{\mathbf{k}}|\alpha_{\mathbf{k}}\rangle$, $\langle\alpha_{\mathbf{k}}|\hat{a}_{\mathbf{k}}^{\dagger} = \alpha_{\mathbf{k}}^{*}\langle\alpha_{\mathbf{k}}|$ and $\bar{n}_{\mathbf{k}} \equiv \langle\alpha_{\mathbf{k}}|\hat{a}_{\mathbf{k}}^{\dagger}\hat{a}_{\mathbf{k}}|\alpha_{\mathbf{k}}\rangle = \alpha_{\mathbf{k}}^{*}\alpha_{\mathbf{k}}$. The last relation means that the average number (expectation value) $\bar{n}_{\mathbf{k}}$ of photons in the coherent state is $\alpha_{\mathbf{k}}^{*}\alpha_{\mathbf{k}}$.

We consider the relationship between the classical electric field $\langle\hat{\mathbf{E}}\rangle$ and the quantized field operator $\hat{\mathbf{E}}$ by using the concept of the coherent state, in which the classical field is defined through $\langle\hat{\mathbf{E}}\rangle = \langle\alpha_{\mathbf{k}}|\hat{\mathbf{E}}|\alpha_{\mathbf{k}}\rangle$. Thus the classical electric field strength is

$$\langle\hat{\mathbf{E}}\rangle = \mathrm{i}\sqrt{\frac{\hbar\omega\mu}{4\epsilon_0 n\gamma V}}[\alpha_{\mathbf{k}}\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r}) - \alpha_{\mathbf{k}}^{*}\exp(-\mathrm{i}\mathbf{k}\cdot\mathbf{r})]\mathbf{e}_{\mathbf{k}}. \tag{31}$$

For convenience, we can assume that the parameter $\alpha_{\mathbf{k}}$ in the coherent state is a real number, and then we have the relation $\alpha_{\mathbf{k}} = \sqrt{\bar{n}_{\mathbf{k}}}$ between the parameter $\alpha_{\mathbf{k}}$ and the total photon number $\bar{n}_{\mathbf{k}}$. Expression (31) is therefore rewritten as

$$\langle \hat{\mathbf{E}} \rangle_{\bar{n}_{\mathbf{k}}} = -\sqrt{\frac{\hbar\omega\mu}{\epsilon_0 n\gamma V}}\sqrt{\bar{n}_{\mathbf{k}}}\sin(kz)\mathbf{e}_{\mathbf{k}}. \tag{32}$$

In the above we use the coherent state to model the classical field. Photon statistics can be discussed in terms of how they vary from that of a coherent state. It should be noted that the classical electric field strength (32) may be discrete since the total number $\bar{n}_{\mathbf{k}}$ of photons in the coherent state is an integer. The degree of discontinuity in $\langle \hat{\mathbf{E}} \rangle_{\bar{n}_{\mathbf{k}}}$ can be defined by

$$\delta = \frac{\langle \hat{\mathbf{E}} \rangle_{\bar{n}_{\mathbf{k}}+1} - \langle \hat{\mathbf{E}} \rangle_{\bar{n}_{\mathbf{k}}}}{\langle \hat{\mathbf{E}} \rangle_{\bar{n}_{\mathbf{k}}+1}}. \tag{33}$$

Further calculation shows that

$$\delta = 1 - \sqrt{\frac{\bar{n}_{\mathbf{k}}}{\bar{n}_{\mathbf{k}} + 1}}. \tag{34}$$

As the number $\bar{n}_{\mathbf{k}}$ of photons increases (and hence the electromagnetic field becomes strong), the degree of discontinuity in the field strength approaches zero ($\delta \to 0$). In other words, the fields become more and more classical (i.e., the field strength $\langle \hat{\mathbf{E}} \rangle_{\bar{n}_{\mathbf{k}}}$ is quasi-continuous) as the average number of photons in the field increases, since the uncertainties become negligible with respect to the amplitude of the electric (and magnetic) fields. In general, the field in a cavity resonator formed by conventional materials is strong (as compared with the electric field $\sqrt{\hbar\omega/\epsilon_0 V}$ of a single photon), and so the field can be regarded as a continuous classical field. However, in a compact subwavelength cavity resonator, the critical electric field strength (the field strength of a single photon) that is a criterion for making a distinction between the discrete quantized field and the continuous classical field is very large, so that a field that even has a fairly large field strength should still be treated as a quantized field since such a field strength may be less than or just a little larger than the critical field strength. This may be interpreted in more details as follows: the expression for the electromagnetic energy density in the dispersive medium takes the form

$$u \equiv \frac{1}{2}\left[\frac{\mathrm{d}(\epsilon\omega)}{\mathrm{d}\omega}\epsilon_0\mathbf{E}^2 + \frac{\mathrm{d}(\mu\omega)}{\mathrm{d}\omega}\mu_0\mathbf{H}^2\right] = \frac{n}{\mu}\frac{\mathrm{d}(n\omega)}{\mathrm{d}\omega}\epsilon_0\mathbf{E}^2. \tag{35}$$

For the gyrotropic chiral material, the parameters ϵ and μ in expression (35) are the equivalent permittivity ϵ_- and permeability μ_- of the eigenmodes. They are defined in expression (27). If quantized, the total electromagnetic energy in a medium with volume V is $uV = (n_{\mathbf{k}}+1/2)\hbar\omega$, where $n_{\mathbf{k}}$ and $\hbar\omega/2$ denote the total number of photons and the zero-point vacuum fluctuation energy. Thus, the electric field strength corresponding to $n_{\mathbf{k}}$ photons of $\mathbf{k}$-mode field (including the vacuum fluctuation energy) is

$$E_{\mathbf{k}} = \sqrt{\frac{\left(n_{\mathbf{k}} + \frac{1}{2}\right)\hbar\omega}{\frac{n}{\mu}\frac{\mathrm{d}(n\omega)}{\mathrm{d}\omega}\epsilon_0 V}}. \tag{36}$$

Then the field strength of a single photon ($n_{\mathbf{k}} = 1$) is given by

$$E_{\mathrm{c}} = \sqrt{\frac{3\hbar\omega}{\frac{2n}{\mu}\frac{\mathrm{d}(n\omega)}{\mathrm{d}\omega}\epsilon_0 V}}, \tag{37}$$

which is a critical field strength to distinguish between the classical and quantized fields. If the field strength is much larger than E_{c}, the field can be considered a classical field. If the field strength has the same order of magnitude as E_{c}, the field is undoubtedly a quantized one. Moreover, the field is a fluctuation field of quantum vacuum if its field strength is less than E_{c}. It is obvious that the critical field strength E_{c} in the compact subwavelength cavity resonator is very large since the volume of the subwavelength cavity resonator containing the left-handed medium can be very small [24]. In particular, the critical field strength E_{c} in the subwavelength resonator formed by the gyrotropic chiral media will be dramatically enhanced by the gyrotropic parameter ϵ_2. By using expressions (27) and (37), one can obtain

$$E_{\mathrm{c}}^{-} = \sqrt{\frac{3\hbar\omega}{2\sqrt{\frac{\epsilon_1-\epsilon_2}{\mu_1-\mu_2}}\frac{\mathrm{d}(n_{-}\omega)}{\mathrm{d}\omega}\epsilon_0 V}}. \tag{38}$$

This is the critical field strength for the eigenmodes inside the subwavelength cavity resonator containing the gyrotropic chiral medium. In order to achieve a negative refractive index, the term $\epsilon_1 - \epsilon_2$ should be small or tend to zero. This, therefore, means that E_{c}^{-} may be very large, and that the quantum vacuum fluctuation field in such a compact cavity resonator is hence very strong. This may enable physicists to study the property of quantum vacuum in the subwavelength cavity resonator. It can be understood that the classical boundary condition for the case of weak fields in a subwavelength cavity resonator cannot be easily fulfilled because the field strength such as (32) is discrete (due to the integer $\bar{n}_{\mathbf{k}}$). This may be utilized to, e.g., the application of so-called manipulation of photon number in the subwavelength cavity resonator.

In addition, the problem of validity of the classical boundary condition in the subwavelength cavity resonator should be considered because of the number–phase uncertainty relation $\Delta n_{\mathbf{k}}\Delta\phi \geqslant \frac{1}{2}$ [26]. Here $\Delta n_{\mathbf{k}}$ and $\Delta\phi$ denote the photon number uncertainty and the phase uncertainty of the photon field. It is known that the number–phase uncertainty relation would imply that a well-defined number state would actually have a phase uncertainty of greater than 2π [26]. Thus the study of the subwavelength cavity resonators with certain length scales is in need for a full quantum treatment. Such a quantum treatment (including the study of the multiphoton state [27] in a compact cavity resonator) is of special interest.

5. Concluding remarks

The chiral material that can exhibit a negative refractive index [19, 18] has been generalized to a case of the gyrotropic chiral medium in this paper. The negative refractive indices of the gyrotropic chiral material have been studied for certain eigenmodes, and the impedances of the eigenmodes have been derived by using the concept of equivalent isotropic media. Since the gyrotropic parameters in the anisotropy matrices of the electric permittivity and magnetic permeability favour the realization of the negative refractive index, the negatively refracting material can be achieved even far off the resonances of the permittivity or permeability. We have shown that the gyrotropic chiral materials can be applied to the subwavelength cavity resonator, and that there may exist a novel effect of field quantization in the subwavelength resonator containing the gyrotropic chiral material with backward wave eigenmodes.

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